

THEORY REFERENCE NX MAGNETICS

This document describes the basics and procedural methods for electromagnetic simulation with the tool NX Magnetism (www.nxmagnetics.de) from the manufacturer Dr. Binde Ingenieure GmbH (www.drbinde.de). In the first chapter, the Maxwell equations on which the simulation is based are presented as well as the resulting application models. In the second chapter, the Laplace equation is used as an example to show in a theoretical way how the numerical solution method works using FEM. In the third and fourth chapters, the two interesting models of direct current simulation and EM simulation in the frequency domain are picked out and considered more from the application-oriented side.

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1 GENERAL PRINCIPLES

1.1 ELECTROMAGNETIC SIMULATION IN GENERAL

1.1.1 GENERAL PRINCIPLE OF ELECTROMAGNETIC FEM

In electromagnetic simulations, the Maxwell equations are solved. Depending on the application, e.g. In electrokinetics or magnetodynamics, the equations are simplified, or parts are omitted. For solving the equations, the finite element method has established itself - along with a few others.

Vector quantities are shown in bold below. The time derivative is written with ∂_t or $\frac{d}{dt}$. The mathematical operators used, *rot* (or *curl*), *grad* and *div*, can usually be found in mathematical formulas.

The Maxwell equations are a set of the following four equations [Bossavit]:

Ampere equation:

$$\text{rot } \mathbf{h} = \mathbf{j} + \partial_t \mathbf{d} \quad (1)$$

Faraday equation:

$$\text{rot } \mathbf{e} = \partial_t \mathbf{b} \quad (2)$$

Conservation of magnetic flux density:

$$\text{div } \mathbf{b} = 0 \quad (3)$$

Conservation of electric flux density:

$$\text{div } \mathbf{d} = \rho_g \quad (4)$$

The symbols and their units mean:

- \mathbf{h} : magnetic field [A/m],
- \mathbf{e} : electric field [V/m],
- \mathbf{b} : magnetic flux density [T],
- \mathbf{d} : electric flux density [C/m²],
- \mathbf{j} : electric current density [A/m²],
- ρ_g : Charge density [C/m³]

In addition, there are the following material equations:

Magnetic relation:

$$\mathbf{b} = \mu \mathbf{h} \quad (5)$$

Dielectric relation:

$$\mathbf{d} = \epsilon \mathbf{e} \quad (6)$$

Ohm-law:

$$\mathbf{j} = \sigma \mathbf{e} \quad (7)$$

The material properties contained therein and their units are:

- μ : Magnetic permeability [H/m]
- ε : Dielectric permeability [F/m]
- σ : Electric conductivity [S/m]

1.1.2 ANALOGY / DIFFERENCES TO STRUCTURAL MECHANICAL FEM

Structural mechanical tasks are also mostly solved with the finite element method. In contrast to electromagnetics, which solves Maxwell's equations, structural mechanics solves equations of elasticity. These are also known as Hook's Law. The primary result is usually displacements at the nodes, with which the translation, rotation and deformation of a solid is described. The nodal displacements are then used to determine strains and, as a result, stresses on the elements.

In electromagnetics, the results of interest are the magnetic and electrical field strengths \mathbf{h} and \mathbf{e} . From these, the other variables such as electrical current density \mathbf{j} , magnetic and electrical flux density \mathbf{b} and \mathbf{d} can be calculated with the help of the material equations. From this, in turn, the other quantities of electric current, magnetic and electric flux can be derived by forming an integral.

Due to the mathematical method used to formulate the Maxwell equations, which will be discussed later, the primary results are not directly \mathbf{h} and \mathbf{e} , but so-called potentials. These potentials are the magnetic vector potential \mathbf{a} and the electrical scalar potential v . The potentials are linked to the quantities that are of interest in the following way:

$$\mathbf{b} = \text{rot } \mathbf{a} \quad (8)$$

$$\mathbf{e} = -\partial_t \mathbf{a} - \text{grad } v \quad (9)$$

By using this potential, the number of unknowns in the equations - and thus the calculation effort - is reduced considerably.

Equation (8) results from the absence of sources of the magnetic field, i.e. from equation (3). Equation (9) can be derived from Faraday's equation and (8).

The potentials themselves can also be represented in post-processing, but are generally to be understood as auxiliary variables that have no direct physical meaning. In some cases, however, a meaning can still be found, e.g. isolines from the magnetic vector potential \mathbf{a} represent the field lines in 2D simulations.

1.1.3 ASSIGNMENT / CLASSIFICATION OF ELECTROMAGNETIC FEM

This section describes the basic properties of some electromagnetic tasks.

1.1.3.1 STATIC PROBLEMS

In the case of static problems, all sizes are independent of time. Therefore, all time derivatives can be omitted in the Maxwell equations.

$$\frac{\partial}{\partial t} = 0$$

1.1.3.2 TIME DEPENDENT PROBLEMS

In the case of time-dependent problems, the magnetic and electric fields are coupled to one another. This can be seen from the Maxwell equations: In the Ampere equation (1) the magnetic field \mathbf{h} is also dependent on the change in the electrical flux density \mathbf{d} over time. In Faraday equation (2), the electric field \mathbf{e} is also dependent on the change in magnetic flux density \mathbf{b} over time.

In the general case, all these time derivatives must be considered. Simplifications are possible on the one hand for slowly changing processes (eddy current and skin effect problems) and on the other hand for processes that change very quickly (wave propagation).

1.1.3.3 EDDY CURRENT AND SKIN EFFEKT PROBLEMS

In the case of slowly changing processes in electrical conductors, the proportions of the conductor current \mathbf{j} clearly outweigh the proportion of the displacement current:

$$\mathbf{j} \gg \partial_t \mathbf{d} \quad (10)$$

Therefore, the ampere equation (1) can be simplified to

$$\text{rot } \mathbf{h} = \mathbf{j} \quad (11)$$

In these cases, eddy currents are considered by dynamic effects. It can be assumed that the entire current moves in a thin layer of the conductor called the skin. The thickness of this layer - also known as the skin depth - is calculated accordingly

$$\delta = \sqrt{\frac{2}{\omega \sigma \mu_0 \mu_r}} \quad (12)$$

therein:

- ω is the natural free frequency, which is connected to the excitation frequency f via the relationship $\omega = 2\pi f$
- μ_0 is the constant vacuum permeability which has the value $4 \cdot \pi \cdot 10^{-7} \text{ H/m}$
- μ_r the magnetic permeability of the area
- σ the electrical conductivity of the area

In the case of dynamic problems, the wavelength in an area often must be determined. This is calculated using the speed of light c and the frequency f

$$\lambda = \frac{c}{f} \quad (13)$$

The speed of light in a vacuum c_0 is known as a constant and is often sufficient for estimations:

$$c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \approx 3e8 \text{ m/s} \quad (14)$$

1.1.3.4 SINUSOIDAL TIME DEPENDENCE

Problems with sinusoidal excitation and linear material properties can be processed very advantageously with the help of complex calculations. The field sizes are specified as pointer sizes. This is shown below for a time-dependent vector \vec{A}

$$\vec{A}(t) = \text{Re}\{\vec{A} \cdot e^{j\omega t}\}$$

Permanent magnets must not be involved here either, because every excitation is assumed to be sinusoidal.

1.2 ELECTROMAGNETIC APPLICATIONS / MODELS IN NX MAGNETICS

This chapter starts with a general discussion of the available models. Then two models are selected that are of particular interest and their theoretical principles and equations are presented.

1.2.1 MODELS / APPLICATIONS IN NX MAGNETICS

This section is based on lecture manuscripts from the University of Lüttig on the subject of “Applied & Computational Electromagnetics” [DularGeuzaine2009], [Geuzaine2012], as well as chapter 6 of the textbook “Simulations with NX” [AnderlBinde2014].

The following figure shows use cases or models that are of interest in electromagnetic simulation and that can arise from Maxwell's equations.

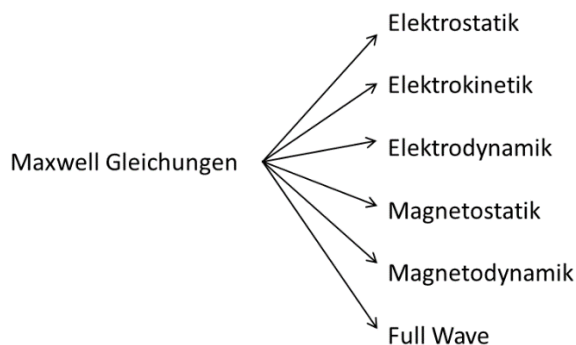


Figure: Electromagnetic use cases and models

The six models that can be derived from Maxwell's equations differ in the way in which they take into account the effects of capacitance, ohmic resistance and inductance. Accordingly, symbols for capacitor, resistor and coil can be assigned:



Electrostatics: Static charges or electrical voltages are specified. As a result, we get the distribution of the electric field. This corresponds to a consideration of the capacitive properties (hence the symbol of a capacitor).



Electrokinetics: (Also called **DC Conduction**) We consider the static distribution of electrical current in conductors. The most important property is electrical conductivity or ohmic resistance (hence the symbol of resistance).



Electrodynamics: This is a combination of electrostatics and kinetics. The distribution of the electric field and electric currents in materials (conductors and non-conductors) is considered. This can also lead to dynamic effects.



Magnetostatics: We consider the static magnetic field that can result from permanent magnets and stationary electrical currents. Because this corresponds to the effect of inductance, we choose the symbol of the coil.



Magnetodynamics: The result is the magnetic field and eddy currents, which result from moving magnets or time-varying currents. The coil with resistance is suitable as a symbol because these two effects are considered.



Full Wave (High frequency): This includes the consideration of complete electromagnetic waves. This requires that all three effects of capacitance, resistance and inductance be considered. This can be used to determine vibrations and resonances, which is why the symbol of the electrical oscillating circuit is suitable.

1.2.2 TWO RELEVANT APPLICATIONS IN DETAIL

Two models / use cases that are of interest are selected and examined in more detail below. These are the applications of direct current simulation (DC Conduction Steady State) and dynamic simulation of the magnetic field in the frequency domain.

1.2.2.1 DC CONDUCTION STEADY STATE

This section deals with the application of direct current simulation. The electromagnetic model for direct current simulation is called electrokinetics. In NX Magnetism, the associated solution name is *DC Conduction Steady State*.

According to [AnderlBinde2014], this model should be selected if

- The wavelength of the exciting frequency is much larger than the component size. This criterion is met in any case if the power sources are stationary.
- Only the stationary flowing electric current can be considered. I.e. only electrically conductive areas are of interest. In particular, the air is not considered.

1.2.2.1.1 PRINCIPLE

The aim of the direct current simulation is to determine the electrical current density and ohmic resistance over a sample structure or between electrodes on the sample.

An example is the application of a circular disc with supply lines, the geometry of which is shown in the next picture.

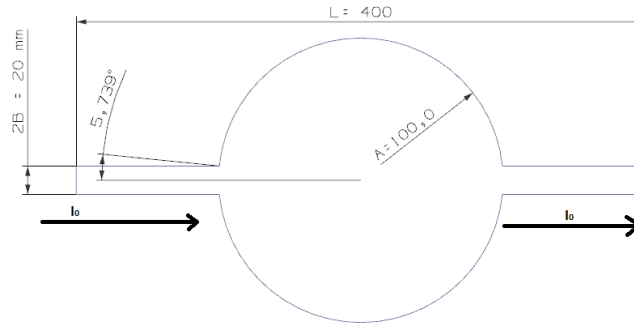


Figure 1 Application of a conductive circular disc with leads for direct current simulation

The ohm resistance that results between the two leads is to be calculated. This task is presented in [Filtz] and [Korolonek].

1.2.2.1.2 RELEVANT PHYSICAL LAWS / EQUATIONS

The Maxwell equations can be simplified for the DC simulation application as described below:

Due to the stationary assumption, all time derivatives are omitted. The descriptive equations result

- Ampere equation:

$$\text{rot } \mathbf{h} = \mathbf{j}$$

it also follows from this

$$\text{div } \mathbf{j} = 0$$

- Faraday equation:

$$\text{rot } \mathbf{e} = 0$$

because of $\text{rot } \mathbf{e} = 0$ a scalar potential v can be introduced with

$$\mathbf{e} = \text{grad } v$$

- Ohm equation:

$$\mathbf{j} = \sigma \mathbf{e}$$

Substituting all three equations together gives the so-called strong formulation of the problem:

$$\text{div } (\sigma \text{ grad } v) = 0 \quad (15)$$

Wegen der Identität [VectorCalculus]

$$\text{div } (\text{grad } f) = \Delta f = \frac{\partial^2}{\partial x^2}$$

this equation can be rewritten with the help of the Laplace operator in such a way that the well-known Laplace equation can be recognized:

$$\sigma \Delta v = 0$$

This equation (17) is the equation to be solved for the application of the direct current simulation.

The equation contains the sought solution v in the second spatial derivative and is therefore called strong. For this reason, it must be converted into a weak formulation for the use of a numerical FEM method. We consider

the development of the weak formulation and the further steps up to the solution in the chapter General procedure.

1.2.2.2 DYNAMIC-SIMULATION OF THE MAGNETIC FIELD IN FREQUENCY DOMAIN

This section deals with the application of dynamic magnetic field simulation in the frequency domain. In NX Magnetics is the associated solution name *Magnetodynamics Frequency*.

According to [AnderlBinde2014] the magnetodynamic model is to be selected if

- The wavelength of the exciting frequency is much larger than the component size. In this case, according to equation (11) the displacement currents can be neglected in the ampere law.
- The skin depth is smaller than the part size. So, there is a pronounced boundary layer and boundary layer effects with eddy currents play a role and must be taken into account.

The magnetodynamic model can be solved in the frequency domain or in the time domain. The frequency solution should only be chosen if

- sinusoidal excitation and
- linear material properties exist and if
- there are no permanent magnets.

1.2.2.2.1 PRINCIPLE

In the dynamic simulation of the magnetic field in the frequency range, the excitation frequency of the electric current or the voltage on the electrode surfaces is specified. As a result, the distribution of the magnetic field and the current distribution come out.

An example is the application of an electrically conductive plate under an AC coil. The geometry is shown in the next picture.

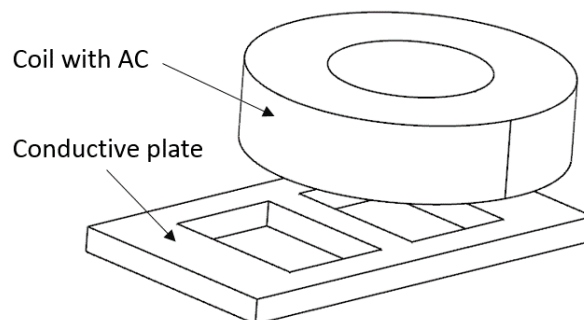


Figure 2 Use case conductive plate under an AC coil for dynamic magnetic field simulation in the frequency range

The aim is to calculate the electric field in the plate that results from the induction of the coil. This task is known under the term *TEAM Problem 3: The Bath Plate* in the literature as a test example for electromagnetic simulations [TEAM3].

1.2.2.2.2 RELEVANT PHYSICAL LAWS / EQUATIONS

The mathematics shown was taken from [Kost]. The Maxwell equations can be simplified as described below for the application dynamic simulation of the magnetic field:

- Ampere equation:

$$\text{rot } \mathbf{h} = \mathbf{j} ,$$

$$\operatorname{div} \mathbf{b} = 0$$

with the vectorpotential \mathbf{a} as

$$\mathbf{b} = \operatorname{rot} \mathbf{a}$$

- Faraday equation:

$$\operatorname{rot} \mathbf{e} = \partial_t \mathbf{b}$$

with the scalarpotential v as

$$\mathbf{e} = -\partial_t \mathbf{a} - \operatorname{grad} v$$

- Ohm equation:

$$\mathbf{j} = \sigma \mathbf{e}$$

- Material law:

$$\mathbf{b} = \mu \mathbf{h}$$

The current \mathbf{j} can be divided into an impressed current \mathbf{j}_s and an eddy current \mathbf{j}_c . Then, the Ohm equation along with Faraday can be written as

$$\mathbf{j} = -\mathbf{j}_s + \mathbf{j}_c = -\mathbf{j}_s + \sigma \mathbf{e} = -\mathbf{j}_s + \sigma(-\partial_t \mathbf{a} - \operatorname{grad} v)$$

Substituting in the ampere equation gives the strong formulation of the problem as

$$\operatorname{rot}(\mu^{-1} \operatorname{rot} \mathbf{a}) + \mathbf{j}_s + \sigma \cdot \partial_t \mathbf{a} + \sigma \cdot \operatorname{grad} v = 0$$

Because of $\operatorname{rot} \operatorname{rot}$, the strong formulation contains the unknown function \mathbf{a} in a dual spatial derivative. In order to develop a weak formulation, the usual procedure is multiplication by the weight or test functions \mathbf{a}' and integration over the area Ω . Further explanations of this method can be found in a subsequent chapter under *Develop formulation*. Written in classic notation results

$$\langle \operatorname{rot}(\mu^{-1} \operatorname{rot} \mathbf{a}), \mathbf{a}' \rangle_{\Omega} + \langle \mathbf{j}_s, \mathbf{a}' \rangle_{\Omega} + \langle \sigma \cdot \partial_t \mathbf{a}, \mathbf{a}' \rangle_{\Omega} + \langle \sigma \cdot \operatorname{grad} v, \mathbf{a}' \rangle_{\Omega} = 0$$

In order to lower the dual $\operatorname{rot} \operatorname{rot}$ derivative by one degree, we use the following Green's identity, which is also called the theorem of the divergence of a vector product or *rot-rot formula* for short

$$\operatorname{div}(\vec{A} \times \vec{B}) = \vec{B} \cdot (\operatorname{rot} \vec{A}) - \vec{A} \cdot (\operatorname{rot} \vec{B})$$

With this formula the dual rot operator on the left can be transformed into a simple one. For this, however, the weight function is given a rot operator. This procedure is also known as *swapping the rot*. It turns out

$$\langle \operatorname{div}(\mu^{-1} \operatorname{rot} \mathbf{a} \times \mathbf{a}') \rangle_{\Omega} + \mu^{-1} \operatorname{rot} \mathbf{a} \cdot \operatorname{rot} \mathbf{a}' \rangle_{\Omega} + \langle \mathbf{j}_s, \mathbf{a}' \rangle_{\Omega} + \langle \sigma \cdot \partial_t \mathbf{a}, \mathbf{a}' \rangle_{\Omega} + \langle \sigma \cdot \operatorname{grad} v, \mathbf{a}' \rangle_{\Omega} = 0$$

Now we deal with the div term. We use the divergence theorem (also called Gauss theorem or Ostrogradsky's theorem), with the help of which the divergence of a volume integral can be converted into the flow over the surface:

$$\langle \operatorname{div}(\vec{A} \times \vec{B}) \rangle_{\Omega} = \langle \vec{n}, \vec{A} \times \vec{B} \rangle_{\partial\Omega}$$

Here \mathbf{n} is the normal vector at the surface. In addition, we use the transformation

$$\vec{n} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{n} \times \vec{A})$$

If this is written together, the result is

$$\begin{aligned} & \langle (\mathbf{n} \times \mu^{-1} \operatorname{rot} \mathbf{a}), \mathbf{a}' \rangle_{\partial\Omega} + \langle \mu^{-1} \operatorname{rot} \mathbf{a}, \operatorname{rot} \mathbf{a}' \rangle_{\Omega} + \\ & \langle \mathbf{j}_s, \mathbf{a}' \rangle_{\Omega_s} + \langle \sigma \cdot \partial_t \mathbf{a}, \mathbf{a}' \rangle_{\Omega_c} + \\ & \langle \sigma \cdot \operatorname{grad} v, \mathbf{a}' \rangle_{\Omega_c} = 0 \end{aligned} \tag{16}$$

the weak form of the magnetodynamic formulation, also known as the a - v form, because potentials a and v are solved according to the unknowns.

Here are the subregions for integration:

- Ω : The whole area
- $d\Omega$: The boundary of the area
- Ω_s : Sub-areas with the direct indication of electrical current
- Ω_c : Sub-areas with electrical conductivity. Eddy currents can arise here.

Further steps leading to the result of a specific task must be carried out in a very similar manner to that described in the next chapter for the general case, i.e. This is followed by the discretization of the area, the setting up of the approximation functions for the elements, setting up the system of equations, inserting boundary conditions, solving the system of equations and the post processing.

2 GENERAL PROCEDURE

The general procedure for the numerical solution using FEM is described in this chapter. Based on a task (step 1), 8 steps are to be carried out:

- Step 1: Task
- Step 2: Develop formulation / choose solution type
- Step 3: Discretization of the area / meshing
- Step 4: Material and physical properties
- Step 5: Establishing the approximation / shape functions for the elements
- Step 6: Set up a system of equations
- Step 7: Insert boundary conditions
- Step 8: Solving the system of equations
- Step 9: Post Processing

These steps are examined in detail below and applied to a one-dimensional example.

2.1 TASK

The general mathematical procedure for the FE analysis of electromagnetic tasks is to be explained in this chapter. The mathematics shown was taken from [Kost]. For the calculation of a specific example, we limit ourselves to the one-dimensional case of a direct current simulation with two different materials. This task was taken over from [Bargallo].

The exemplary differential equation to be solved is the Laplace equation (15), which we derived in Section 1.2.2.1 . This describes i.e. the application of electrokinetics or direct current simulation. It reads as follows:

$$\operatorname{div}(\sigma \operatorname{grad} v) = 0 \quad (17)$$

In the specific case of the one-dimensional direct current simulation, this can be written in this way:

$$\frac{d}{dx} \left(\sigma \frac{dv}{dx} \right) = 0 \quad x \in (0, L) \quad (18)$$

Here, v is the unknown function (e.g. the electrical potential in the case of direct current simulation), σ is a known parameter (e.g. the electrical conductivity).

The boundary conditions are generally divided into Dirichlet and Neumann conditions. With Dirichlet the unknown is determined directly and with Neumann its spatial derivation is determined. One or the other boundary condition must be applied to the entire edge of the area. We define the Dirichlet condition on the boundary Γ_1 and the Neumann condition on Γ_2 :

$$v = \bar{v}$$

$$q_0 = \frac{d v}{d n}$$

For our concrete example of the one-dimensional direct current simulation, we define Dirichlet conditions at both limits and omit the Neumann condition:

$$v(x = 0) = v_0 \quad (19)$$

$$v(x = L) = 0 \quad (20)$$

2.2 DEVELOP FORMULATION / SELECT SOLUTION TYPE

In this step, the governing physical equations are set up in such a way that they can be solved using numerical methods. The starting point is the strong formulation of equation (17)

$$\operatorname{div}(\sigma \operatorname{grad} v) = 0$$

or for our one-dimensional example

$$\frac{d}{dx} \left(\sigma \frac{dv}{dx} \right) = 0 \quad x \in (0, L)$$

In this strong formulation, the unknown quantity v is contained in the second derivative. In the numerical method, the quantity v is to be approximated by element-wise shape functions. In order to be able to use linear shape functions as well, v in the equation must be reduced by one order. This then corresponds to a weak formulation. The development of a weak formulation is usually done by weighting (multiplying) with test functions, integrating over the arithmetic domain and applying a Green identity, which ensures that the test functions are increased by one degree and the functions sought are reduced by one. This method of developing a weak shape is also called the *method of weighted residuals*. We want to describe them in more detail:

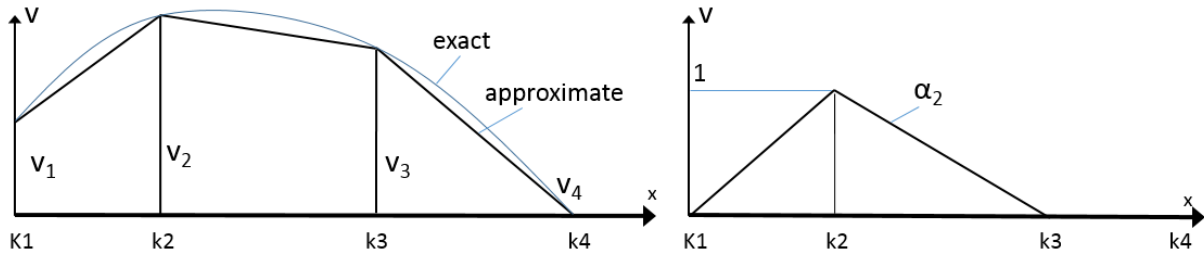
When changing from the exact to the approximate solution, an error or residual R must be allowed, i.e.

$$\operatorname{div}(\sigma \operatorname{grad} v) = R$$

The unknown solution v can be approximated by a sum of discrete node values v_k , each weighted with a weighting function (also called shape function, test function or basis function) a_k , i.e. be multiplied:

$$v = \sum_{k=1}^p \alpha_k \cdot v_k$$

Such a linear shape function for the one-dimensional case, for example for node 2, is shown in the next figure on the right. The approximate and the exact solution are shown on the left. With this approach, only the discrete node values v_k must be set accordingly in order to obtain an approximate solution.



So that the residual can be small, we allow it to be multiplied by weighting functions (test functions) that are named w . To minimize the residual, it is integrated over the area and set to zero. This means that it is made to disappear. This is also called orthogonalizing the residual to its weighting functions [Bargallo]. Hence it results

$$\int_{\Omega} R w \, d\Omega = \int_{\Omega} [\text{div}(\sigma \, \text{grad} \, v)] w \, d\Omega = 0$$

with initially arbitrary weight functions w . This idea is the core of the numerical method and the method has not only proven itself for the FEM, but also for other numerical methods such as the Finite Volume Method (FVM) or the Boundary Element Method (BEM). In the way in which the weighting function w is chosen, various approximation techniques differ from which we use the *Galerkin* method: Here, the same shape functions are used for the weighting of the residual as for the unknown solution v . I.e. later we equate the shape functions:

$$w_l = \alpha_l$$

It should be noted here that two groups of weighting functions are now involved: One for the unknown solution v and one for the residual R . The group for the residual must be understood in such a way that it results in individual equations for each node. The overall system of equations will later be put together from these equations.

This method of weighted residuals in connection with the Galerkin method is universal and will be pursued further below. Alternatively, another method is known as the variation method (also known as the variation integral or Rayleigh-Ritz method), which usually leads to the same solution. With this method, a calculation formula for the energy (LAGRANGE energy) of the electromagnetic field is established. The calculation rule for this energy is called the energy functional. It is required that this energy is minimal and this is achieved by deriving the energy according to all degrees of freedom and setting it to zero. This creates a linear system of equations for the degrees of freedom. In [Kost] it is shown that the same systems of equations arise from the variation method and the Galerkin method.

In order to reduce the second derivative in the unknown function v , the first GREEN sentence is used. This says

$$\int_{\Omega} U_1 \Delta U_2 \, d\Omega = - \int_{\Omega} \text{grad} \, U_1 \, \text{grad} \, U_2 \, d\Omega + \int_{\Gamma} U_1 \, \text{grad} \, U_2 \, \vec{n} \, d\Gamma$$

The first GREEN sentence can also be written in a form that is named in [DularGeuzaine2009] as the *grad div Green formula*. This identity can also be found in [VectorCalculus]:

$$(\mathbf{u}, \text{grad} \, v')_{\Omega} + (\text{div} \, \mathbf{u}, v')_{\Omega} = (\mathbf{n} \, \mathbf{u}, v')_{\partial\Omega}$$

When writing this and some of the following equations, the classic notation is chosen, which is often used for weak formulations with test functions. Here $(,)_{\Omega}$ means an integration over Ω and the comma means a multiplication. Vectors are marked either with an arrow or in bold.

Put together it results

$$\int_{\Omega} (\sigma \text{grad } v) \text{grad } w \, d\Omega - \int_{\Gamma} \sigma w \frac{dv}{dn} \, d\Gamma = 0 \quad (21)$$

The boundary integral represents the Neumann boundary condition. In many practical cases $\frac{dv}{dn} = 0$ and thus this integral is omitted (then a homogeneous Neumann condition is present) otherwise it must be incorporated into the equation system by adding it to the right-hand side is added. The Dirichlet boundary condition can later be inserted into the equation system simply by setting v to the known values

If these equations are now written together and a homogeneous Neumann boundary condition is assumed, then the sum of all n elements finally results

$$\sum_{i=1}^n \sum_{k=1}^p \left[\int_{\Omega^i} \sigma \text{grad } \alpha_k \text{grad } \alpha_l \, d\Omega \right] v_k = 0, \quad l = 1 \dots p \quad (22)$$

In order to create an equation system with p equations for p nodes, p weight functions (shape functions) α_l must be introduced one after the other. The counter k therefore runs over all nodes. The counter i on the other hand only runs over those nodes that have an influence on the respective node k , i.e. across all neighboring nodes. In this way, the desired weighting is possible at node k .

If the matrix elements K_{lk} resulting from the product $\text{grad } \alpha_k \text{grad } \alpha_l$ and the integration over the individual elements Ω^i are combined to a matrix \mathbf{K} and the unknown node potentials v_k to a vector \mathbf{U} , the linear system of equations is created

$$\mathbf{K} \cdot \mathbf{U} = 0$$

These matrix elements are specifically written down in a later section.

2.3 DISCRETISATION OF THE AREA / MESHING

In this step the area is divided into small sub-areas, i.e. finite elements. This can be done in several ways. 0D, 1D, 2D and 3D elements are possible.

For our example task we choose three 1D line segments that subdivide the area $(0, L)$. Both elements and nodes are numbered, as shown in the figure below. The different materials can also be seen with $\sigma_{1,2}$

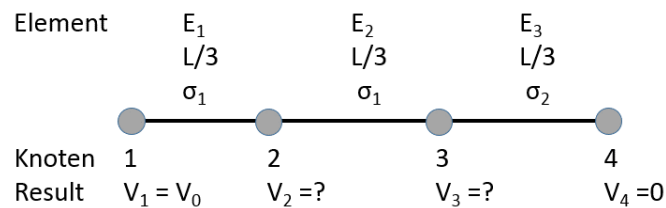


Figure 3 Discretization of the one-dimensional sample task

2.4 MATERIAL- AND PHYSICAL PROPERTIES

Depending on the physical equation to be solved, material parameters and other physical properties must be specified. These properties can be isotropic, i.e. the same in all spatial directions, or orthotropically, i.e. with different values in the three spatial directions. In addition, these quantities can also be functions of temperature or other quantities.

In the case of direct current simulation (DC Conduction Steady State), only the electrical conductivity in the elements is required. For models with a magnetic field, the magnetic permeability and the magnetic remanence field strength in the case of permanent magnets are required. If the electric field is calculated, the electric permittivity is required accordingly [Hilzinger-Rodewald].

For our example we want to assume two different electrical conductivities σ_1 and σ_2 . In addition, the conductivity should be isotropic and $\sigma_1 = 2\sigma_2$.

The electrical conductivity, as well as other material parameters, can be defined as a scalar value or as a vector in the three spatial directions. When specified as a vector, it is possible to use local, regional coordinate systems for this. In this case, transformation calculations from the respective local to the global coordinate system are required.

A regional definition with orthotropic electrical conductivity is shown below.

The following figure shows the example of the circular disk with the area divided into connectors (bold) and the circle (thin).

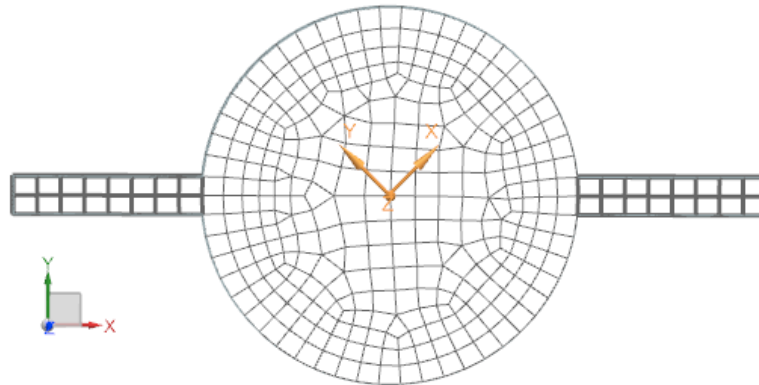


Figure 4 Example of the circular disk with orthotropic material properties

Let the circular area be defined orthotropically with the central coordinate system

$$\sigma_{local} = \begin{pmatrix} \sigma_{x\ local} \\ \sigma_{y\ local} \end{pmatrix}$$

The transformation matrix that results from the local coordinate system rotated by the angle φ is given by

$$RotMatrix = \begin{pmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The conductivity in global coordinates can now be specified as

$$\sigma_{global} = RotMatrix^T \cdot \sigma_{local} \cdot RotMatrix$$

It should be noted that the global conductivity has now become a 9 component tensor.

2.5 SET UP THE SHAPE FUNCTIONS FOR THE ELEMENTS

In this step, the shape functions are set up with which the unknown solution is approximated. These can be linear or higher-order functions. They can also be assigned to nodes, edges or element areas.

These assignments differ from solution type to solution type in NX Magnetics. Some are shown below:

- 2D/3D Electrostatics: Nodal degrees of freedom for potential v
- 2D/3D Magnetostatics and -dynamics: Edge degrees of freedom for potential \mathbf{a}
- 3D Magnetostatics and -dynamics: Nodal degrees of freedom for potential v
- 2D/3D Electrokinetics (DC current): Nodal degrees of freedom for potential v

The degrees of freedom can be the magnetic potentials \mathbf{a} and v or temperature T or mechanical displacement \mathbf{u} . Linear, quadratic or even higher shape functions are possible.

For this one dimensional example we choose linear functions, which are described below.

In every e -th element, the unknown v should be approximated by

$$v^e(x) = a^e + b^e x.$$

Here a^e and b^e are the constants to be found. This unknown should now be expressed by the node variables. For linear elements, there are two nodes on each element: one at x_1^e and one at x_2^e . Inserted above, there are two expressions for the two unknown node variables:

$$v_1^e(x) = a^e + b^e x_1^e$$

$$v_2^e(x) = a^e + b^e x_2^e$$

Solve for a^e and b^e gives

$$b^e = \frac{v_2^e - v_1^e}{l^e}$$

$$a^e = v - \frac{v - v_1^e}{l^e} \cdot x_1^e$$

$$l^e = x_2^e - x_1^e$$

Inserted into the above approach function results

$$v^e(x) = \left(\frac{x_2^e - x}{l^e} \right) v_1^e + \left(\frac{x - x_1^e}{l^e} \right) v_2^e = \sum_{j=1}^2 \alpha_j^e(x) \cdot v_j^e$$

$$\alpha_1^e(x) = \frac{x_2^e - x}{l^e}$$

$$\alpha_2^e(x) = \frac{x - x_1^e}{l^e}$$

The two functions α are also called basis functions. These are shown graphically in the following picture.

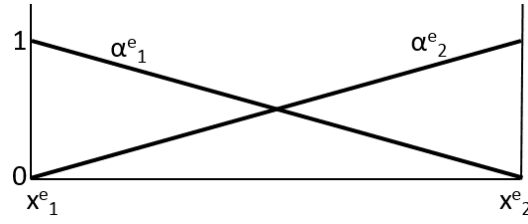


Figure 5 Representation of the shape functions in a one-dimensional element

In an FEM program, the shape functions are not set up new for each local element, but only once for a reference element. This shape function can then be applied to every global element by means of a suitable transformation. For the transformation, a Jacobi matrix J is set up for each element. Details are e.g. read in [ZienkiewiczTaylor] or [Bathe].

2.6 SET UP SYSTEM OF EQUATIONS

In this step of the FEM, the system of equations is set up that results from the formulation and the shape functions.

In the case of our one-dimensional example from equation (18), the components K_{ij} for an element e from equation (22) result according to the rule

$$K_{ij}^e = \int_{x_1^e}^{x_2^e} \sigma \frac{d\alpha_i^e}{dx} \cdot \frac{d\alpha_j^e}{dx} dx$$

This can be calculated analytically, and the coefficients result for a single element e

$$K_{11}^e = K_{22}^e = \frac{\sigma}{l^e}$$

$$K_{12}^e = K_{21}^e = -\frac{\sigma}{l^e}$$

Our one-dimensional problem has three elements and four nodes. This results in a 4 x 4 matrix for the overall system K . For each of the three elements, the small 2 x 2 matrix mentioned above is created, which is inserted into the overall matrix. The three individual element matrices and how they are inserted into the overall matrix are shown below

$$K^1 = \begin{pmatrix} K_{11}^1 & K_{12}^1 & 0 & 0 \\ K_{21}^1 & K_{22}^1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, K^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & K_{11}^2 & K_{12}^2 & 0 \\ 0 & K_{21}^2 & K_{22}^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, K^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & K_{11}^3 & K_{12}^3 \\ 0 & 0 & K_{21}^3 & K_{22}^3 \end{pmatrix}$$

The element matrices are added in the overall matrix, i.e. our overall matrix and the system of equations result as

$$\begin{pmatrix} K_{11}^1 & K_{12}^1 & 0 & 0 \\ K_{21}^1 & K_{22}^1 + K_{11}^2 & K_{12}^2 & 0 \\ 0 & K_{21}^2 & K_{22}^2 + K_{11}^3 & K_{12}^3 \\ 0 & 0 & K_{21}^3 & K_{22}^3 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (23)$$

A symmetrical K matrix is established that is only occupied on a narrow band of the main diagonal. This property is very advantageous for the fast and memory-efficient solution of the system with the help of modern linear equation solvers. This property of the K matrix comes from the fact that the shape functions for a node always only include those other nodes that are adjacent.

2.7 INSERT BOUNDARY CONDITIONS

Let us first deal with the Dirichlet boundary conditions. In order to insert the first boundary condition for our example from equations (19) $v_1 = v_0$ we change the first line of the equation system (23) as follows

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ K_{21}^1 & K_{22}^1 + K_{11}^2 & K_{12}^2 & 0 \\ 0 & K_{21}^2 & K_{22}^2 + K_{11}^3 & K_{12}^3 \\ 0 & 0 & K_{21}^3 & K_{22}^3 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} v_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

But now the symmetry of the K matrix has been lost. The symmetry can be restored with the following change:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & K_{22}^1 + K_{11}^2 & K_{12}^2 & 0 \\ 0 & K_{21}^2 & K_{22}^2 + K_{11}^3 & K_{12}^3 \\ 0 & 0 & K_{21}^3 & K_{22}^3 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} v_0 \\ -K_{21}^1 \cdot v_0 \\ 0 \\ 0 \end{pmatrix}$$

After inserting the second boundary condition $v_4 = 0$ and restoring the symmetry, the result is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & K_{22}^1 + K_{11}^2 & K_{12}^2 & 0 \\ 0 & K_{21}^2 & K_{22}^2 + K_{11}^3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} v_0 \\ -K_{21}^1 \cdot v_0 \\ -K_{12}^3 \cdot 0 \\ 0 \end{pmatrix}$$

This system of equations has only two unknowns and can therefore be written as

$$\begin{pmatrix} K_{22}^1 + K_{11}^2 & K_{12}^2 \\ K_{21}^2 & K_{22}^2 + K_{11}^3 \end{pmatrix} \cdot \begin{pmatrix} v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} -K_{21}^1 \cdot v_0 \\ 0 \end{pmatrix}$$

We come to Neumann boundary conditions. If e.g. at $x = L$ a Neumann boundary condition would exist like

$$\frac{d v}{d n} = q_0$$

so it can be seen from equation (21) that this has to be inserted on the right side of the equation system, i.e. the system of equations (23) would be changed as follows

$$\begin{pmatrix} K_{11}^1 & K_{12}^1 & 0 & 0 \\ K_{21}^1 & K_{22}^1 + K_{11}^2 & K_{12}^2 & 0 \\ 0 & K_{21}^2 & K_{22}^2 + K_{11}^3 & K_{12}^3 \\ 0 & 0 & K_{21}^3 & K_{22}^3 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ q_0 \end{pmatrix}$$

2.8 SOLVING THE SYSTEM OF EQUATIONS

The system of equations to be solved or the K matrix is usually a

- sparsely populated, diagonal dominant,
- symmetrical,
- positive definite matrix.

Iterative or direct solution methods can be used for this type of matrix. Both types are available in NX Magnetics, whereby the direct solver is used by default.

For the solution of our one-dimensional example, the coefficients of the element matrices are inserted

$$K_{11}^e = K_{22}^e = \frac{\sigma}{l^e} , \quad K_{12}^e = K_{21}^e = -\frac{\sigma}{l^e}$$

It results for the entire system

$$\begin{pmatrix} \frac{\sigma_1}{l^e} + \frac{\sigma_1}{l^e} & -\frac{\sigma_1}{l^e} \\ -\frac{\sigma_1}{l^e} & \frac{\sigma_1}{l^e} + \frac{\sigma_2}{l^e} \end{pmatrix} \cdot \begin{pmatrix} v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} \frac{\sigma_1}{l^e} \cdot v_0 \\ 0 \end{pmatrix}$$

We assume that $\sigma_1 = 2\sigma_2$ in the example. Then the potentials arise as primary solutions as

$$v_2 = \frac{3}{4} v_0$$

$$v_3 = \frac{1}{2} v_0$$

2.9 POST PROCESSING

The last step is post processing. Here, the sought values are derived from the primary solved variables. In the case of our example, the potentials at the nodes are now known and the currents at the nodes are to be calculated. In our example, Ohm's law

$$\mathbf{j} = \sigma \mathbf{e}$$

and the formula for the potential

$$\mathbf{e} = \text{grad } v$$

are used. According to

$$\mathbf{j} = \sigma \mathbf{e} = \sigma \text{grad } v = \sigma \frac{dv}{dx}$$

a current density is first calculated on the three elements. It turns out

$$j_1 = \sigma_1 \frac{v_2 - v_1}{L/3} = \sigma_1 \frac{3}{4} \frac{v_0}{L}$$

$$j_2 = \sigma_1 \frac{v_3 - v_2}{L/3} = \sigma_1 \frac{3}{4} \frac{v_0}{L}$$

$$j_3 = \sigma_2 \frac{v_4 - v_3}{L/3} = \sigma_2 \frac{3}{2} \frac{v_0}{L} = \sigma_1 \frac{3}{4} \frac{v_0}{L}$$

The current I itself is calculated by integrating the current density over the cross-sectional area. If the area is s , then is

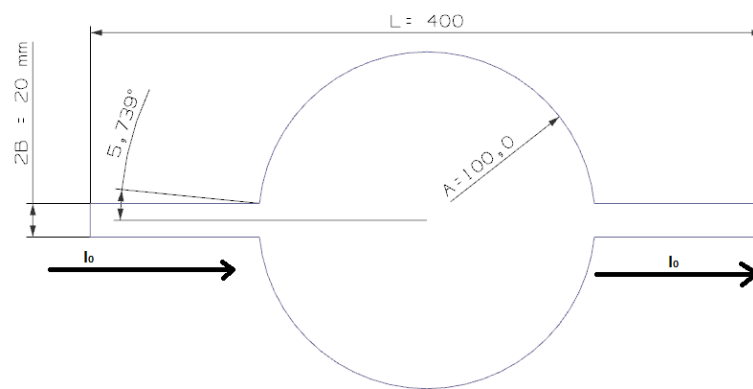
$$I_1 = I_2 = I_3 = \int j_1 ds = \sigma_1 \frac{3}{4} \frac{v_0}{L} s$$

3 DIRECT CURRENT – SIMULATION (DC CONDUCTION STEADY STATE)

After the theoretical and numerical background has been clarified in Chapter 2, this chapter will examine direct current simulation from the application-oriented side. I.e. a concrete example is to be solved using the NX Magnetics system. In the procedure, we stick to the 8 steps that are necessary for the general procedure for the EM simulation.

3.1 TASK

The geometry from the following figure is given (see also Figure 1). A constant electric current should flow through the two electrode surfaces on the left and right.



The thickness is 20 mm. The material is copper with the material properties

- Electrical conductivity of $\sigma = 58 \text{ S/m}$ (isotropic)

The objectives of the simulation are:

- The determination of the current density and its distribution in the sample structure and
- Determination of the ohmic resistance over the sample structure or the defined electrode surfaces.

3.2 DEVELOP FORMULATION / SELECT SOLUTION TYPE

In this task, stationary currents are to be simulated in an electrical conductor. The excitation frequency is zero. Therefore, the model of direct current simulation is chosen. The corresponding formulation for solving the Laplace equation is activated by selecting the *DC Conduction Steady State* solution type in NX Magnetics. For the sake of generality, we choose a 3D simulation, although 2D would also be possible and in this case would be quite useful.

The following figure shows the associated setting in NX Magnetics.

Solution	
Name	Solution 1
Solver	MAGNETICS
Analysis Type	3D Electromagnetics
2D Solid Option	None
Solution Type	DC Conduction Steady State

DC Conduction Steady State	
Output Requests	Plot
Time Steps	<input type="checkbox"/> Scalarpotential <input type="checkbox"/> Electric Fieldstrength <input checked="" type="checkbox"/> Current Density <input type="checkbox"/> Eddy Current Losses Density
Coupled Thermal	Table
	<input checked="" type="checkbox"/> Ohm Resistance <input type="checkbox"/> Electrode Voltage <input type="checkbox"/> Electrode Current <input type="checkbox"/> Eddy Current Losses

Figure 6 Solution type settings for DC simulation

3.3 DISCRETISATION OF THE AREA / MESHING

The discretization of the area is carried out through automatic meshing in the NX system. The creation of meshes is carried out in the FEM file. 2D and 3D elements are possible.

A mesh independence study should be carried out to check the quality of the mesh. I.e. a second calculation should be carried out with a slightly different element size (e.g. half). If the results of the two calculations differ only slightly (e.g. difference <1%), the quality of the mesh has been proven.

The various types of elements available for selection are described below. For our example we want to choose tetrahedra.

3.3.1 TETRAHEDRA ELEMENTES

Tetrahedral elements can be used in 3D solutions. They can be generated automatically on all 3D solids and are therefore used universally. Such a mesh is shown in the next figure.

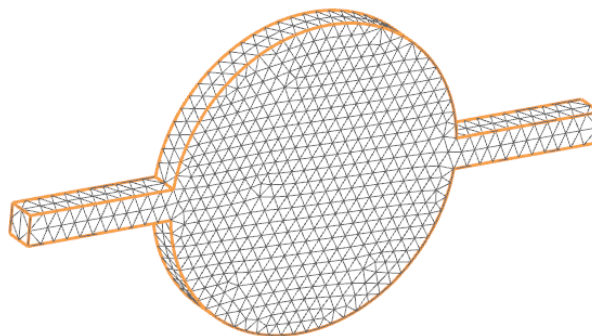


Figure 7 Tetrahedra meshing on the circular disk

Tetrahedral elements combine the advantages of ease of creation and good accuracy. In terms of accuracy, they behave a little worse than hexahedron elements. I.e. with the same number of degrees of freedom, hexahedra are somewhat closer to the theoretically exact result.

3.3.2 HEXAHEDRA ELEMENTS

Hexahedral elements can be used in 3D solutions. They can only be created on extrudable geometries, which significantly limits their usability. The example geometry can be extruded and a hexahedron meshing is shown in the next figure.

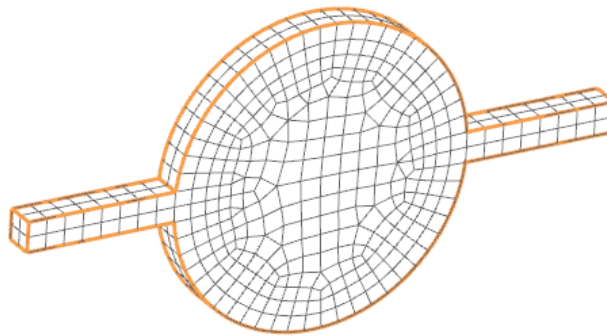


Figure 8 Hexahedron meshing on the circular disk

Compared to tetrahedra, hexahedron elements achieve higher accuracy with the same number of degrees of freedom.

3.4 MATERIAL- AND PHYSICAL PROPERTIES

The only material property required in this DC simulation is electrical conductivity. This is defined either isotropically or orthotropically in NX.

- isotropically definition:

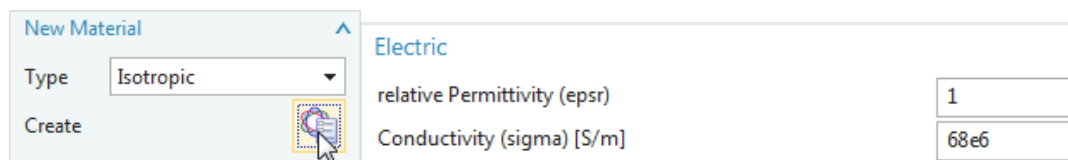


Figure 9 Specification of an isotropic electrical conductivity

- orthotropically definition:

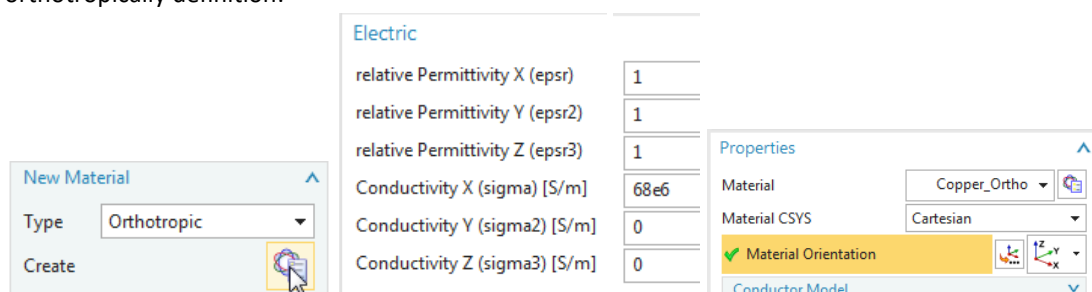


Figure 10 Specification of an orthotropic electrical conductivity with a local coordinate system

We choose isotropic properties for the example.

3.5 SET UP OF THE SHAPE FUNCTIONS FOR THE ELEMENTS

In the 2D / 3D direct current simulation (DC Conduction Steady State) nodal degrees of freedom are used. Linear (Element Order: First) and quadratic (Element Order: Second) shape functions are possible. The type of shape function can be selected in NX Magnetix in the solver parameters, as shown in the next figure.

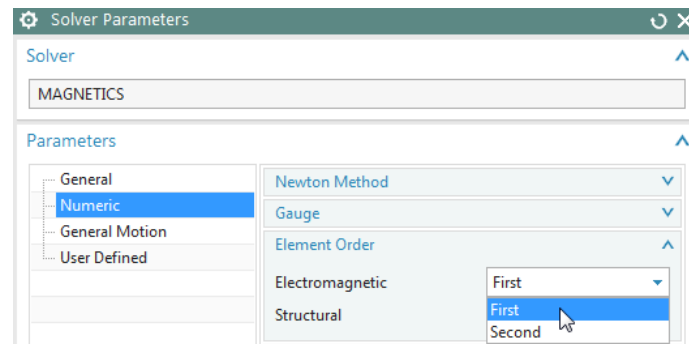


Figure 11 Setting the type of shape functions

For our example, linear shape functions are to be chosen.

3.6 SET UP SYSTEM OF EQUATIONS

No user action is required to set up the system of equations in NX Magnetix.

3.7 INSERT BOUNDARY CONDITIONS

Boundary conditions or loads in direct current simulations (DC Conduction Steady State) mean either the specification of electrical voltages or electrical current at the boundaries of the area.

The definition of voltage boundary conditions can be done as a potential difference. The associated electrical current is established between different voltage potentials according to the electrical conductivity and geometry of the structure and is calculated as the result of the simulation. This type of boundary condition leads to simple definitions in the system of equations and has already been presented theoretically in Section 2.7.

For the definition of enforced electric current or voltage boundary conditions, a method is used that is presented under [Dular1999-1]. This method allows the definition of current in the same simple way as voltage or potential. A special feature here is that current and voltage are associated with the electrical conductor as global quantities on all electrode surfaces and are available with high accuracy, even with coarse meshes.

In the case of our task, a current should be specified that flows between the two electrode surfaces. The following options are available for this:

- Generate the current through a potential difference between the electrode surfaces. However, the size of the difference is not known.
- By two current conditions with opposite signs on the two electrode surfaces.
- By a voltage definition with zero volts on one electrode and a current definition with the desired size on the other electrode.

The most common way is the third: a zero voltage and a current condition. This is to be done with this example. The next figure shows the geometry with these two conditions.

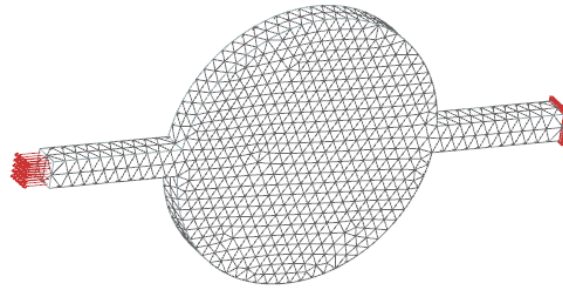


Figure 12 Boundary conditions for direct current simulation: zero voltage on the right and current condition on the left electrode surface

3.8 SOLVING THE SYSTEM OF EQUATIONS

The system of equations is solved by executing the Solve function. After solving, the primary quantity is available, i.e. the electrical scalar potential v is available for the subsequent post processing.

3.9 POST PROCESSING

The last step is post processing. The primary variable v is further processed according to the output requirements and displayed as a plot or table result.

In the case of our direct current simulation, the current density is calculated as shown under 2.9. This is output as a plot result and is shown in the following figure.

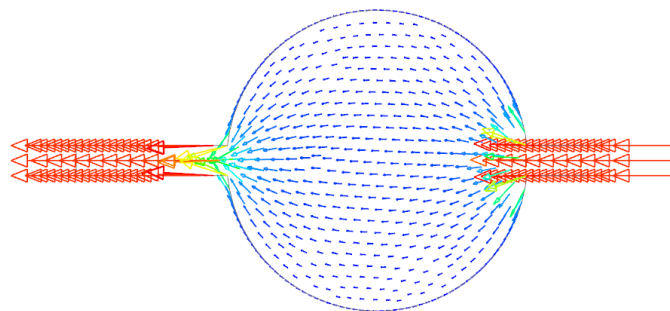


Figure 13 Current density distribution from the example task for direct current simulation

As a further result, the Ohm resistor was requested. Because the voltage and the current were calculated as integral quantities on all electrode surfaces and are available, the ohm resistance can easily be calculated using the formula

$$R = \frac{U}{I}$$

calculated by the program.

This example results in an ohm resistance of 1.07e-5 ohms. An analytical calculation of this problem can be found in [Korolonek] and gives a value of 1.081e-5 Ohm.

4 EM-SIMULATION IN FREQUENCY DOMAIN

In this chapter a concrete example for the EM simulation in the frequency domain (magnetodynamic frequency) is to be solved. As in chapter 3 with the direct current simulation, we assume that the theoretical and numerical backgrounds are clarified in chapter 2 and that the application-oriented side can be examined here. In the procedure, we stick to the 8 steps for the general procedure for EM simulation.

4.1 TASK

There is a plate with two cutouts made of electrically conductive material and a coil arranged above it, which is operated with alternating current [TEAM3]. See the following illustration.

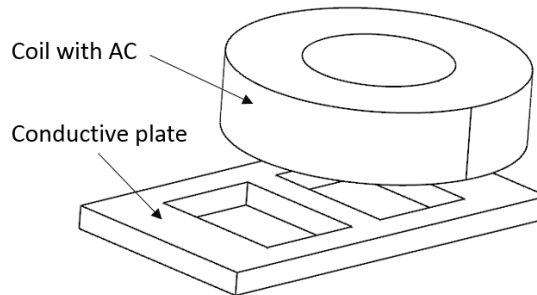


Figure 14 Geometry of the sample task for EM simulation in the frequency domain

The aim of the simulation is the investigation

- the flux density at defined points / areas (FE nodes),
- the field of the induced eddy currents in the plate,
- the field of eddy current losses in the sample plate,
- the total eddy current losses in the entire sample plate.

The dimensions can be found in [TEAM3]. This task is based on the following additional information:

- turns in the coil: 126
- Current in one turn: Wechselstrom mit 10 A (Amplitude)
- Excitation frequency: 50 Hz
- Electrical conductivity of the coil: 58e6 S/m (isotrop)
- Rel. magnetic permeability of the coil: 1
- Electrical conductivity of the plate: 32760000 S/m (isotropic)
- Rel. magnetic permeability of the plate: 1

4.2 DEVELOP FORMULATION / SELECT SOLUTION TYPE

In this task, time-harmonic (sinusoidal) current sources are to be simulated, which lead to a time-dependent, changing magnetic field and induced eddy currents. Therefore a dynamic model has to be chosen.

It must be checked whether a high-frequency model (full wave) must be selected or whether the magnetodynamic model that is suitable for low and medium frequencies is enough. For this purpose, the wavelength λ is determined according to equation (13). It turns out

$$\lambda = \frac{c}{f} = \frac{3e8 \text{ m/s}}{50 \text{ Hz}} = 6000000 \text{ m}$$

The wavelength is compared with the component size L . Because $\lambda > L$, the electromagnetic wave cannot develop in the component and it is enough to choose the magnetodynamic model.

Now it must be checked whether the simulation is carried out in the frequency domain or in the time domain. A time domain simulation is much more computationally expensive because of the individual time steps. It would therefore be advantageous to choose the frequency domain. Chapter 1.2.2.2 shows the criteria for simulation in the frequency domain. We check these now:

- Sinusoidal excitation: Criterion met
- linear material properties: Criterion met
- no permanent magnets: Criterion met

Therefore, the dynamic model can be chosen in the frequency domain.

This model and the corresponding formulation are activated by selecting the Magnetodynamic Frequency solution type in NX Magnetics.

The following figure shows the associated settings in NX Magnetics

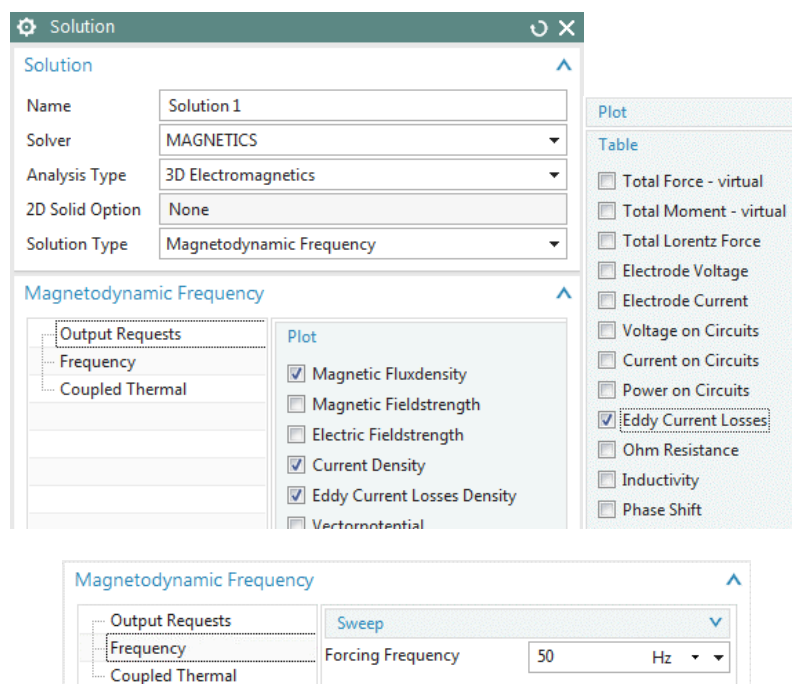


Figure 15 Settings in NX Magnetics for the EM simulation in the frequency domain

This figure also shows the corresponding output requirements resulting from the objectives of this simulation. The forced excitation frequency of 50 Hz must be created as a modeling object in register *Frequency* (not shown in the figure).

4.3 DISKRETISATION OF THE AREA / MESHING

In addition to the plate and the coil, the air must also be meshed because the magnetic field must be considered here. In addition, an infinity mesh should be used.

4.3.1 CAD PREPARATIONS AND MESH-MATING-CONDITIONS

The CAD geometry includes an area for the air, which is often created as a sphere. The components are subtracted from this air area in CAD. *Mesh mating conditions* with the *Glue-Coincident* setting are created in the FEM file, which ensure that the subsequent meshes will have identical nodes at the material boundaries.

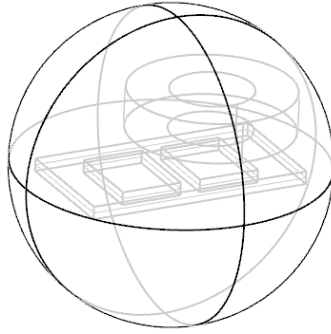


Figure 16 Representation of the air volume and the subtracted components

At the boundary of the volume, a condition is chosen later that forces the field to be tangential. Therefore should

- either the volume of air is chosen so large that the field lines can develop in it almost unhindered,
- or infinity elements are used.

In our case we want to make use of the second option - the infinity elements.

4.3.2 MESHING THE PLATE WITH CONSIDERING THE SKIN DEPTH

For problems with eddy currents, the skin depth δ in electrically conductive areas must always be considered. In this boundary area of the conductive structure, which is characterized by the skin depth, the meshing must have at least one, better 2 or up to 5 element layers. This is necessary in order to enable the eddy currents to develop in the simulation. It is also helpful if this boundary layer area is meshed in layers with hexahedron elements. The skin depth in the plate is calculated according to equation (12) as

$$\delta = \sqrt{\frac{2}{\omega \sigma \mu_0 \mu_r}} = \sqrt{\frac{2}{2\pi \cdot 50\text{Hz} \cdot 32760000 \frac{\text{S}}{\text{m}} \cdot 4\pi 10^{-7} \cdot 1}} \approx 12,4 \text{ mm}$$

At 6.35 mm, the plate thickness is smaller than the skin depth. Therefore, the elements in the plate do not have to be particularly fine, because a pronounced thin eddy current layer is not to be expected. It should be noted that the skin depth becomes smaller as the frequency increases.

In this example, the plate is meshed with hexahedra, because this is possible due to the geometry. Tetrahedral elements would also be possible without any problems. The mesh is shown in the next figure.

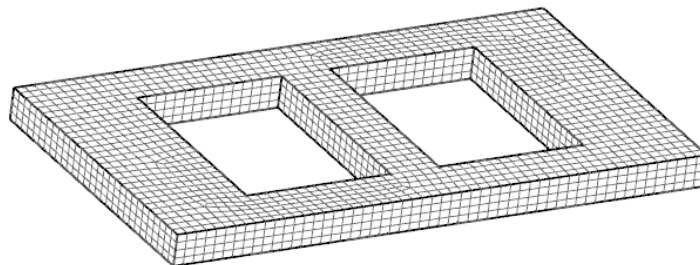


Figure 17 Representation of the cross-linking of the sample plate

4.3.3 MESHING THE COIL WITH DEFINITION OF THE STRANDED DIRECTIONS

Hexahedra are also suitable for meshing the coil, whereby tetrahedra would also be suitable. The mesh can be created without any further special features and there are also no high demands on the fineness, because no results are required in the coil itself. The mesh is shown in the next figure on the left.

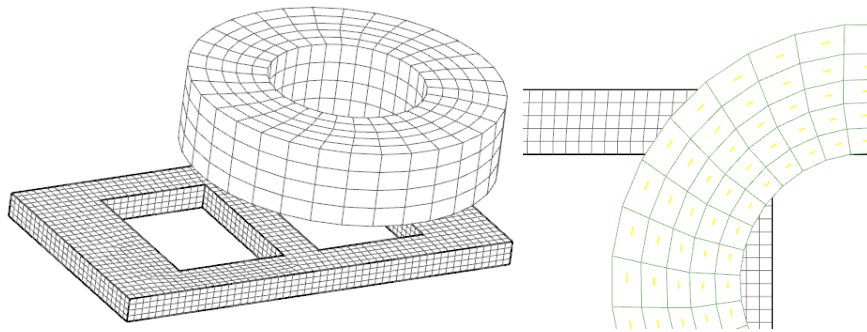


Figure 18 On the left the representation of the network of the coil. Right the winding directions

Because it is a wound coil with 126 turns, it is necessary that the directions of the turns are communicated to the mesh. This is done by specifying a guide curve, in the direction of which each element of the coil determines a direction vector. The guide curve can be a circle, as in our example. However, it can also be a spline curve with any curvature that describes the direction of the windings on bobbins with a complex shape.

The guide curve is defined in the coil mesh with the aid of the *Mesh Associated Data* function. In our example, one of the circle edges is selected. With the *Preview* button, the direction of each element can be checked as shown in the previous figure on the right.

4.3.4 MESHING THE AIR WITH HEX-TET-TRANSITION

The next step is the meshing of the air. Due to the geometry, this must be done with tetrahedral elements. The transition from the already existing hexahedra to tetrahedra should take place via pyramid elements. This way enables correct processing due to the degrees of freedom used on element edges.

The pyramid elements are created as a separate mesh that can also be viewed. The following figure shows this on the left. The rest of the air mesh is shown on the right.

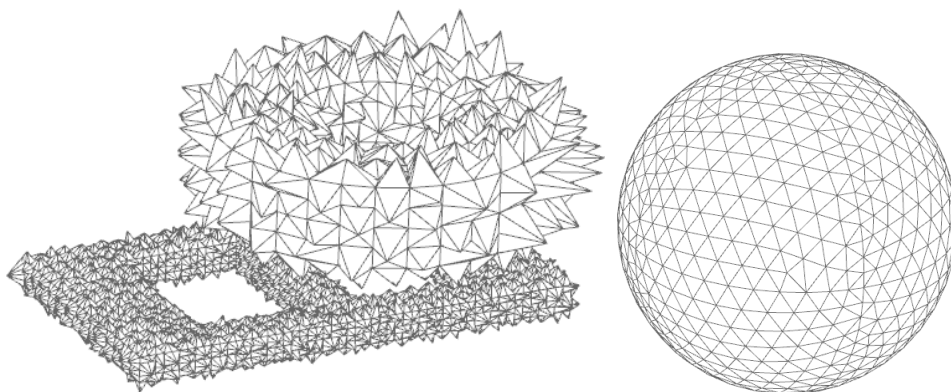


Figure 19 On the left the representation of the pyramid elements for the Hex-Tet transition. On the right the air mesh.

4.3.5 MESHING WITH INFINITY ELEMENTS

The use of infinity elements is helpful when the electromagnetic field to be simulated can spread infinitely in the air. To keep the simulation model small, the meshing in the simulation should only include an air area that

is close to the geometry of interest. In order to capture the infinitely large airspace nonetheless, infinity elements can be used. These elements use the same shape functions as other elements. The only difference is in the transformation from the local coordinate system into the global one with the help of the Jacobi matrix J. Here, the outer edge of the layer of infinity elements is transformed into infinity. This corresponds to a distortion of the calculation area in this area.

The following figure shows this effect: Without infinity elements on the left and with infinity elements on the right. The area with infinity elements is the outer layer with 3 quad elements. On the left the field lines are compressed because they have to remain in the airspace that is too small. On the right, they can expand realistically. This of course has a strong effect on the field strength in the area of interest.

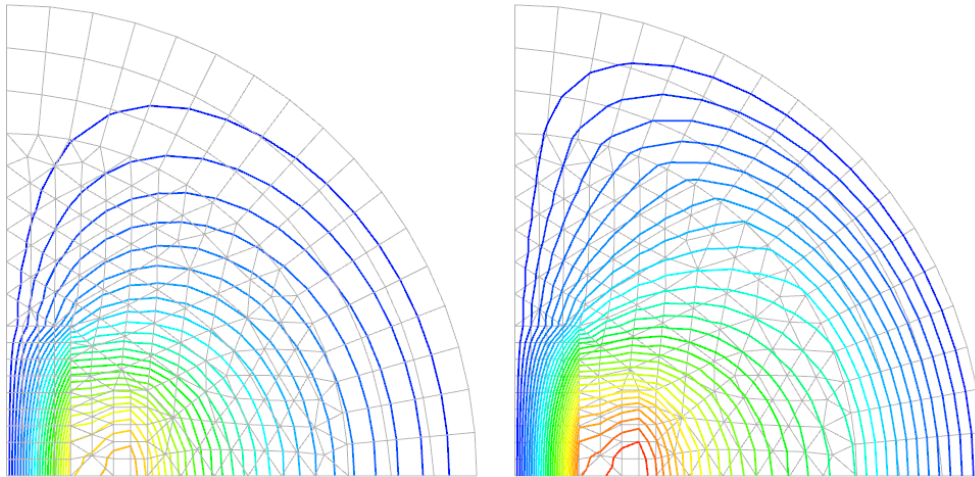


Figure 20 Field lines at the core with a coil model. Left without infinity elements, right with.

Now we come to the concrete creation of these special elements in NX Magnetics.

The infinity elements are created in an additional layer on the outside of the air volume. If infinity elements are used, the air must be either a sphere or a cuboid.

If the air is available as a 3D sphere, the 3D layer for the infinity elements can be created in a very simple way by extrusion of the outer element surfaces. This is shown for our example in the next figure.

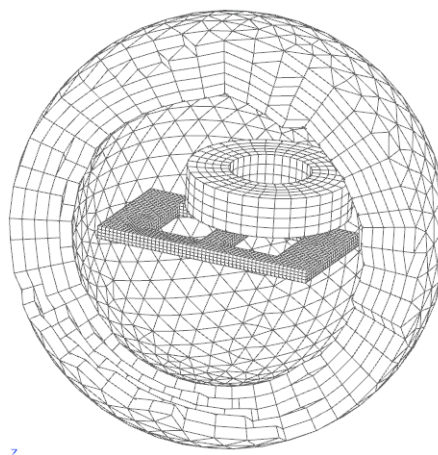


Figure 21 A layer around the air sphere with infinity elements is shown. For better visibility, an opening is shown and the air sphere is not shown.

As a rule of thumb, the layer thickness of the infinity elements should be around a third of the radius of the air sphere. In this layer there should be about 5 to 10 elements above the thickness.

The parameters of these infinity elements are defined in the next section.

4.4 MATERIAL- AND PHYSICAL PROPERTIES

Next, the material and physical parameters are defined. This is done in the *Mesh Collectors* and the *Physical Property Tables* (physicals for short) contained in the FEM file.

The areas coil, plate and air are defined with the usual physical of the preset type *SolidPhysical*. Only the infinity elements are defined with a physical of the *Infinity3D* type, which must be activated in the mesh collector of the mesh.

4.4.1 COIL

The properties of the coil are *Material* and a *Stranded* conductor model.

The required material parameters are

- relative Permeability = 1
- Electric Conductivity = 58e6 S/m

The required parameters for the *Conductor Model* are

- Model: *Stranded, Vectors defined in Mesh Associated Data*
- Number of Turns: 126
- Fillfactor: 1
- Coil Section Area: 400 mm²

4.4.2 PLATE

The properties of the plate are *Material* and the preset *Massive* conductor model.

The required material parameters are

- relative Permeability = 1
- Electric Conductivity = 32760000 S/m

4.4.3 AIR

The air receives the following material properties

- relative Permeability = 1
- Electric Conductivity = 0 S/m

Zero conductivity is required for air to be recognized as non-conductive.

4.4.4 INFINITY ELEMENTS

The infinity elements are given the following parameters:

- Physical Property Type: *Infinity3D*

- Shape: *Spherical Shell*
This type is suitable for use on spherical volumes.
- Inner Radius at Shell: 75 mm
Here the radius at the boundary of the air sphere to the infinity elements is entered.
- Shell Thickness: 30 mm
The entire thickness of the infinity layer is entered here.

4.5 SET UP THE SHAPE FUNCTION FOR THE ELEMENTS

No information from the user is required for setting up the shape functions. In the solver parameters, a distinction can be made between first (linear) and second (quadratic) order, whereby we want to calculate with first order.

4.6 SET UP THE SYSTEM OF EQUATIONS

No information from the user is required to set up the system of equations.

4.7 INSERT BOUNDARY CONDITIONS

A constraint of the type *Zero Potential - Flux tangent* is required at the boundary of the calculation area. With this condition, the potential \mathbf{a} on the boundary surface is set to zero. With the Rand term $\langle (\mathbf{n} \times \mu^{-1} \text{rot } \mathbf{a}), \mathbf{a}' \rangle_{\text{d}\Omega}$ from equation (16) results, that the magnetic field \mathbf{h} is directed tangentially at the boundary and is equal to zero.

As a second constraint, a load is applied to the coil which directly defines the given electrical current, i.e. it is inserted in equation (16) in the term $\langle \mathbf{j}_s, \mathbf{a}' \rangle_{\Omega_s}$

4.8 SOLVING THE SYSTEM OF EQUATIONS

To solve the system of equations, the user only needs to execute the *Solve* function.

4.9 POST PROCESSING

The primary result of the calculation is the two potentials \mathbf{a} and v .

- The desired magnetic flux density can be calculated using equation (8) as follows

$$\mathbf{b} = \text{rot } \mathbf{a}$$

- The eddy current distribution in the plate results from equation (9) and the Ohm equation $\mathbf{j} = \sigma \mathbf{e}$ to

$$\mathbf{j} = \sigma \mathbf{e} = \sigma (-\partial_t \mathbf{a} - \text{grad } v)$$

- The field of eddy current losses P (Eddy Current Losses) results from the eddy current distribution through the relationship

$$P = \frac{\mathbf{j}^2}{\sigma} = \sigma \cdot (-\partial_t \mathbf{a} - \text{grad } v)^2$$

- The total eddy current losses in the entire sample plate result from the integration of the eddy current losses in the plate geometry.

5 SOURCES

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